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Majorana Neutrino Masses from Anomalous $U(1)$ Symmetries

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Abstract

We explore the possibility of interpreting the solar and atmospheric neutrino data within the context of the Minimal Supersymmetric Standard Model augmented by a single $U(1)$ anomalous family symmetry spontaneously broken by non–zero vacuum expectation values of a pair of singlet fields. The symmetry retains a dimension-five operator which provides Majorana masses for left-handed neutrino states. Assuming symmetric lepton mass matrices, the model predicts inverse hierarchical neutrino mass spectrum, $\theta_{13} = 0$ and large mixing while at the same time it provides acceptable mass matrices for the charged fermions.

1.   Introduction

Recent neutrino oscillation data [1, 2] imply that neutrino squared mass differences are tiny, with $\Delta m_\odot^2 \approx 10^{-5}eV^2$ and $\Delta m_{atm}^2 \approx 10^{-3}eV^2$. Moreover, atmospheric neutrino mixing is rather maximal ($\theta_{atm.} \approx \frac{\pi}{4}$), while the corresponding solar neutrino mixing is large. These experimental facts suggest that the Yukawa couplings related to neutrino masses are highly suppressed compared to those of quarks and charged leptons while their mixing is much larger than that of the quark sector.

Large mixing may indicate an underlying structure of the mass matrix determined by a symmetry beyond the Standard Model gauge group. A natural candidate would be an additional $U(1)$ family symmetry that is broken at some high scale M, a scenario proposed some time ago for the explanation of the charged fermion mass hierarchy [3, 4, 5, 6, 7]. Several theoretical proposals have been put forward in the last few years to interpret neutrino data by additional family symmetries [8, 9, 10, 11, 12, 13]. These attempts are also motivated by the fact that the majority of string models constructed so far include several (possibly anomalous) additional $U(1)$'s.

It is interesting therefore to explore whether a simple extension of the Standard Model (SM) gauge symmetry may predict an approximate form of the leptonic mixing matrix. Our aim is to provide a solution using only the minimal fermion spectrum of the supersymmetric version of the Minimal Supersymmetric Standard Model (MSSM). Thus, we will interpret the experimental data, without introducing the right-handed neutrinos. Indeed, this is possible since recent data can be well fitted with left-handed neutrinos alone as in the context of MSSM, there exists a single operator[14, 15][1] suppressed by one power of mass which can provide Majorana masses for all three neutrinos. This lepton number violating operator has the form

$$\frac{\zeta_\nu^{a\beta}}{M}(\bar{L}_a^{c\,i}H^j\epsilon_{ji})(H^lL_\beta^k\epsilon_{lk}) \;\; \equiv \;\; \frac{\zeta_\nu^{a\beta}v^2}{M}\bar{\nu}_{La}^c\nu_{L\beta} \tag{1}$$

where, $\zeta_\nu^{a\beta}$ is an effective Yukawa coupling depending on the details of the theory, v is a Higgs vacuum expectation value (vev) which is of the order of the electroweak scale and M stands for a large scale that will turn out to be of the order 10^{13-14} GeV. This scale is quite low to be identified with the GUT or the string scale in the context of the heterotic string theory, however, it is compatible with the effective gravity scale in theories with large extra dimensions obtained in the context of Type I string models. [2]

In the present work we explore the possibility that neutrino masses and mixing can be interpreted with the help of an additional anomalous $U(1)$ family symmetry which at the same time is responsible for the generation of charge fermion mass hierarchy. This symmetry could be anomalous and anomaly cancellation is assumed to happen in the context of a fundamental theory valid above the scale M. We show that in a generic model an additional abelian symmetry can account for atmospheric data and predicts $\theta_{13} = 0$. We also show how secondary effects possibly arising from additional singlet(s) or some

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[1]For recent reviews see also [16, 17]
[2]For a similar argument, see also [18]

alternative mechanism, as supersymmetry breaking, can under certain assumptions render the model compatible with all recent experimental data. We finally derive explicit charge assignments that reproduce the above results.

2. Description of the Model

We consider the MSSM with gauge symmetry $G_{SM} = SU(3) \times SU(2)_L \times U(1)_Y$ as an effective field theory below a scale M of a fundamental theory. In the context of the G_{SM} symmetry, all gauge invariant Yukawa terms relevant to quark and charged lepton masses appearing at the tree-level superpotential are

$$\mathcal{W} = y_{ij}^u Q_i U_j^c H_2 + y_{ij}^d Q_i D_j^c H_1 + y_{ij}^e L_i E_j^c H_1. \tag{2}$$

In the case of models constructed in the framework of string theory, there are explicit examples where the MSSM fields are charged under (at least) one additional abelian anomalous $(U(1)_X)$ factor that prevents terms not invariant under this symmetry from appearing in (2). Usually, the appearance of the additional $U(1)_X$ symmetry is accompanied by at least a pair of MSSM singlets $(\Phi, \bar{\Phi})$ with opposite $U(1)_X$-charges. Φ and $\bar{\Phi}$ can acquire vevs leading to the breaking of the extra abelian symmetry.

Assuming natural values of the Yukawa couplings λ_{ij} in (2) (i.e., order one), and taking into account the observed low energy hierarchy of the fermion mass spectrum, we infer that only couplings associated with the third generation should remain invariant at tree-level. Mass terms for the lighter fermions are to be generated from higher order non-renormalizable superpotential couplings. Such higher order invariants are formed by adding to the non-invariant tree-level coupling an appropriate number of $U(1)_X$-charged singlet fields which compensate the excess of the $U(1)_X$-charge. In the case supersymmetric models, the magnitudes of the singlet vevs $\langle\Phi\rangle$ and $\langle\bar{\Phi}\rangle$ are related by the D-flatness conditions of the superpotential, while perturbative considerations require that the vevs for the singlet fields are about one order of magnitude below the effective theory scale M scale, therefore lighter generations couplings will be suppressed by powers of $\lambda, \bar{\lambda}$ where

$$\lambda = \frac{\langle\Phi\rangle}{M}, \, \bar{\lambda} = \frac{\langle\bar{\Phi}\rangle}{M} \tag{3}$$

Introducing the generic charge $U(1)_X$-charge assignments of Table 1, the charges of the entries of the corresponding mass matrices are

$$C_{ij}^u = q_i + u_j \, , \, C_{ij}^d = q_i + d_j \, , \, C_{ij}^e = \ell_i + e_j. \tag{4}$$

Restricting the analysis to the investigation of symmetric fermion mass matrices we obtain the following constraints

$$\begin{aligned}
q_i + u_j &= q_j + u_i \\
q_i + d_j &= q_j + d_i \\
\ell_i + e_j &= \ell_j + e_i.
\end{aligned} \tag{5}$$

Fermion	Charge	Higgs	Charge
$Q_i(3, 2, \frac{1}{6})$	q_i	$H_1(1, 2, -\frac{1}{2})$	h_1
$D_i^c(\bar{3}, 1, \frac{1}{3})$	d_i	$H_2(1, 2, \frac{1}{2})$	h_2
$U_i^c(\bar{3}, 1, -\frac{2}{3})$	u_i		
$L_i(1, 2, -\frac{1}{2})$	ℓ_i	$\Phi(1, 1, 0)$	$+1$
$E_i^c(1, 1, 1)$	e_i	$\bar{\Phi}(1, 1, 0)$	-1

Table 1: $U(1)_X$ charge assignments for MSSM fields. The $U(1)_X$ charges of the two extra singlet fields Φ and $\bar{\Phi}$, are taken to be $+1$ and -1 respectively.

Moreover, the requirement that the third generation mass couplings appear at tree-level imposes the additional constraints

$$
\begin{aligned}
q_3 + u_3 + h_2 &= 0 \\
q_3 + d_3 + h_1 &= 0 \\
\ell_3 + e_3 + h_1 &= 0.
\end{aligned}
\tag{6}
$$

Since in our configurations the top, bottom and τ–Yukawa couplings are equal at the high scale M, up to order one coefficients, the difference between the top mass (m_t) and the bottom mass (m_b) must arise mainly from a large Higgs vev ratio $\tan\beta = \frac{v_2}{v_1} \gg 1$. The case of small $\tan\beta \sim \mathcal{O}(1)$ can also be worked out easily in a similar way, by modifying conditions (6) so that the $b - \tau$ Yukawa couplings appear at a higher order.

From the above we conclude that the general form of the superpotential couplings contributing to the fermion mass matrices, are divided into two categories:
a) The tree-level couplings

$$
\mathcal{W}_{tree} = y_{33}^u Q_3 U_3^c H_2 + y_{33}^d Q_3 D_3^c H_1 + y_{33}^e L_3 E_3^c H_1
\tag{7}
$$

with $y_{u,d,e}$ being the order-one Yukawa couplings, and
b) non-renormalizable contributions of Yukawas allowed by the $GS \times U(1)_X$ gauge symmetry. For the up, down quarks and charged leptons these are

$$
\mathcal{W}_{n.r.}^{(1)} \propto Q_i U_j^c H_2 \varepsilon^{C_{ij}^u} + Q_i D_j^c H_1 \varepsilon^{C_{ij}^d} + L_i E_j^c H_1 \varepsilon^{C_{ij}^l}
$$

where, C_{ij}^a $(a = u, d, l)$ are defined in (4) and ε is defined as follows

$$
\varepsilon^k = \begin{cases} \lambda^k & \text{if } k = [k] < 0 \\ \bar{\lambda}^k & \text{if } k = [k] > 0 \\ 0 & \text{if } k \neq [k] \end{cases}
\tag{8}
$$

where $[k]$ stands for the integer part of k. As far as neutrinos are concerned, these are massless at tree-level, however, the non-renormalizable mass term (1) leads directly to a light Majorana mass matrix involving only the left handed components ν_{Lj}

$$
\mathcal{W}_{n.r.}^{(2)} = \frac{\zeta_\nu^{\alpha\beta}}{M} \varepsilon^{C_{ij}^\nu} (\bar{L}_a^{c\,i} H_2^j \epsilon_{ji})(H_2^l L_\beta^k \epsilon_{lk}) \equiv \zeta_\nu^{\alpha\beta} \varepsilon^{C_{ij}^\nu} \frac{v_2^2}{M} \bar{\nu}_{La}^c \nu_{L\beta}
\tag{9}
$$

with $v_2 = \langle H_2 \rangle \approx O(m_W)$ and $C_{ij}^\nu = 2h_2 + \ell_i + \ell_j$.

Conditions (5),(6) imply that the $U(1)_X$-charges of the up and down quark entries are equal. Furthermore, the quark charge-entries can be written only in terms of two combinations, namely $q_1 - q_3$ and $q_2 - q_3$. In addition to the conditions (5),(6), in order to obtain acceptable quark mass matrices we further need to impose [7]

$$q_1 - q_3 = \frac{n}{2} \ , \ q_2 - q_3 = \frac{m}{2} \ \text{ where } m + n \neq 0, \ m, n = \pm 1, \pm 2, \ldots \tag{10}$$

thus, quark matrices depend only on the two integers m, n. [3]

The corresponding entries of the quark matrices take the form

$$C_q = C_d = \begin{pmatrix} n & \frac{m+n}{2} & \frac{n}{2} \\ \frac{m+n}{2} & m & \frac{m}{2} \\ \frac{n}{2} & \frac{m}{2} & 0 \end{pmatrix} \tag{11}$$

Similarly for leptons we define the parameters $2n' = l_1 - l_3$ and $2m' = l_2 - l_3$, where m', n' are integers and the associated $U(1)_X$–charge matrix takes the form

$$C_e = \begin{pmatrix} n' & \frac{m'+n'}{2} & \frac{n'}{2} \\ \frac{m'+n'}{2} & m' & \frac{m'}{2} \\ \frac{n'}{2} & \frac{m'}{2} & 0 \end{pmatrix} \tag{12}$$

The zero charge in the position 33 of the above charge-matrices is due to the fact that we demand the appearance of the corresponding Yukawa couplings at the tree-level superpotential. For the remaining entries, a proper power of the appropriate expansion parameter is needed.

We can re-express the generic fermion charges of Table 1 in terms of the new parameters which we choose to be m, n, m', n' that appear in the quark and charged lepton matrices and q_3, ℓ_3, h_2, h_1. The resulting assignments are presented in Table 2.

The $U(1)_X$-charge entries for the light Majorana neutrino mass matrix take the form

$$C_\nu = \begin{pmatrix} n' + \mathcal{A} & \frac{m'+n'}{2} + \mathcal{A} & \frac{n'}{2} + \mathcal{A} \\ \frac{m'+n'}{2} + \mathcal{A} & m' + \mathcal{A} & \frac{m'}{2} + \mathcal{A} \\ \frac{n'}{2} + \mathcal{A} & \frac{m'}{2} + \mathcal{A} & \mathcal{A} \end{pmatrix} \tag{13}$$

where we have introduced the new parameter

$$\mathcal{A} = 2(l_3 + h_2) \tag{14}$$

We observe that the neutrino $U(1)_X$–charge entries differ from the corresponding charged leptonic entries by the constant $\mathcal{A}$

$$C_{ij}^\nu = C_{ij}^e + \mathcal{A} \tag{15}$$

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[3]In the context of heterotic string theory, anomaly cancellation conditions imply further relations between q_i and ℓ_i [4]. These relations impose further constraints on the $U(1)_X$ charges [7].

field	generation		
	1	2	3
Q	$\frac{n}{2} + q_3$	$\frac{m}{2} + q_3$	q_3
U^c	$\frac{n}{2} - q_3 - h_2$	$\frac{m}{2} - q_3 - h_2$	$-h_2 - q_3$
D^c	$\frac{n}{2} - q_3 - h_1$	$\frac{m}{2} - q_3 - h_1$	$-h_1 - q_3$
L	$\frac{n'}{2} + l_3$	$\frac{m'}{2} + l_3$	l_3
E^c	$\frac{n'}{2} - l_3 - h_1$	$\frac{m'}{2} - l_3 - h_1$	$-h_1 - l_3$
Higgs			
H_1	h_1	H_2	h_2

Table 2: Fermion $U(1)_X$ charge assignments after introducing the integer parameters m, n and m', n' that appear in the quark and charge lepton matrices respectively.

The entries of the charged lepton mass matrix C^e_{ij} are integers or half-integers, therefore, in order to obtain non-zero entries in the Majorana mass matrix too, the parameter $\mathcal{A}$ has to be either integer or half-integer. If $\mathcal{A}$ is an integer, an additional condition should be satisfied to insure mixing effects in the neutrino sector. Indeed, if both C^ν_{ij} and C^e_{ij} charge entries have the same sign, then the corresponding mass matrix elements are proportional by the same proportionality factor $\varepsilon^{\mathcal{A}}$, thus both matrices can be diagonalised simultaneously and the leptonic mixing matrix equals to the identity. Nevertheless, in this case, we could obtain mixing effects if some of the charge matrix elements satisfy the condition $C^\nu_{ij} \cdot C^e_{ij} \le 0$. Then, according to (8) the charged lepton and neutrino mass matrices involve different expansion parameters, thus they are no longer proportional. On the contrary, there are no constraints if $\mathcal{A}$ is half integer and we will analyze this case in the sequel as it is more promising.

3. Neutrino Masses and Mixing

In this section we search for explicit $U(1)_X$ charge assignments for MSSM particles that provide phenomenologically acceptable mass textures for all MSSM fermions and in particular for neutrinos. The basic structure of the mass matrices and mixing angles which meet the phenomenological requirements can be obtained without referring to a set of particular $U(1)_X$-charges. Explicit examples with sets of charges for all fermion and Higgs fields will be given in the end of this section. Before we present viable cases, we should note that our procedure exhibits here the basic structure of the mass matrices and mixing. The most striking feature, is that the extension of the G_{SM} symmetry to include an $U(1)_X$ anomalous factor can reproduce the correct hierarchy of all fermion fields while at the same time the recent neutrino oscillation data are interpreted to a good approximation by a lepton mixing matrix involving two mixing angles, one originating from the charged leptonic matrix matrix and the second by the light Majorana mass matrix. However, at this level of analysis the value of the non-vanishing coefficients of the Yukawa superpotential terms are unknown, since their calculation requires a detailed knowledge of the fundamental theory above the scale M (possibly string theory). Hence, in the present analysis, we restrict ourselves in the description of the general characteristics of the theory, which are

nevertheless very interesting.

We first note that in our framework the quark mass matrix depends only on m, n while the leptonic one depends on m', n'. We can thus fix the parameters m, n, so that a correct hierarchical quark mass spectrum is obtained. The lepton sector can be then worked out independently, choosing appropriate values for the two additional parameters m' and n'.

In terms of the l parameter defined in (8) and up to order-one coefficients the quark mass matrices take the form

$$M_{u,d} \sim m_0^{u,d} \begin{pmatrix} \varepsilon^n & \varepsilon^{\frac{m+n}{2}} & \varepsilon^{\frac{n}{2}} \\ \varepsilon^{\frac{m+n}{2}} & \varepsilon^m & \varepsilon^{\frac{m}{2}} \\ \varepsilon^{\frac{n}{2}} & \varepsilon^{\frac{m}{2}} & 1 \end{pmatrix} \tag{16}$$

This matrix has been work out in detail in the past [4, 5, 7] and it is known to ensure the hierarchical mass structure for a variety of m, n pairs.

Next, in order to obtain a viable set of lepton mass matrices and mixing, a systematic search shows that the charge parameters m', n' should be $n' =$ odd , $m' =$ even. Under this choice the charged lepton mass matrix takes the form

$$M_e = m_0^e \begin{pmatrix} \delta\,\varepsilon^{n'} & 0 & 0 \\ 0 & \varepsilon^{m'} & \alpha\,\varepsilon^{\frac{m'}{2}} \\ 0 & \alpha\,\varepsilon^{\frac{m'}{2}} & 1 \end{pmatrix} \tag{17}$$

where we have explicitly introduced two (out of three) order-one parameters a and δ that account for the Yukawa couplings and renormalization effects. The lepton mass eigenvalues are

$$m_e = m_0^l\,\delta\,\varepsilon^{n'}, \; m_\mu = m_0^l(1 - a^2)\varepsilon^{m'}, \; m_\tau = m_0^l(1 + a^2\varepsilon^{m'}) \tag{18}$$

Introducing $\tan(2\phi) = 2\,a\,\varepsilon^{\frac{m'}{2}}/(1 - \varepsilon^{m'})$ the diagonalising matrix of the charged lepton sector takes the form

$$V_l(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}. \tag{19}$$

Turning to the neutrino sector the Majorana neutrino mass matrix takes the form

$$M_\nu^0 = m_0^\nu \begin{pmatrix} 0 & -\varepsilon^{\frac{m'+n'}{2}+\mathcal{A}} & \zeta\,\varepsilon^{\frac{n'}{2}+\mathcal{A}} \\ -\varepsilon^{\frac{m'+n'}{2}+\mathcal{A}} & 0 & 0 \\ \zeta\,\varepsilon^{\frac{n'}{2}+\mathcal{A}} & 0 & 0 \end{pmatrix} \tag{20}$$

where ζ stands for an order one coefficient. This mass matrix can be diagonalised by a unitary matrix $V_\nu(\omega)$, where $\tan\omega = \zeta\,\varepsilon^{-m'/2}$, and can lead to bimaximal mixing in the case that the two mass matrix elements are equal [10].

Solution A				Solution B			
field	generation			field	generation		
	1	2	3		1	2	3
Q	4	2	0	Q	4	2	0
D^c	2	0	-2	D^c	1	-1	-3
U^c	4	2	0	U^c	4	2	0
L	$\frac{9}{4}$	$-\frac{1}{4}$	$-\frac{5}{4}$	L	$\frac{9}{4}$	$-\frac{1}{4}$	$-\frac{5}{4}$
E^c	$\frac{11}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	E^c	$\frac{7}{4}$	$-\frac{3}{4}$	$-\frac{7}{4}$
Higgs				Higgs			
H_1	2	H_2	0	H_1	3	H_2	0
Singlets				Singlets			
Φ	1	Φ	-1	Φ	1	Φ	-1

Table 3: Examples of $U(1)_X$ charges which lead to the neutrino mass matrix structure discussed in the text.

The neutrino mass eigenvalues are $m_{\nu_1} = -m_0^{\nu'}, m_{\nu_2} = m_0^{\nu'}$ and $m_{\nu_3} = 0$, with $m_0^{\nu'} = m_0^\nu \, \varepsilon^{\mathcal{A}+\frac{m'+n'}{2}} \sqrt{1 + \zeta^2 \varepsilon^{-m'}}$ giving at this level for the mass square differences

$$\Delta m_{atm}^2 = \Delta m_{23}^2 = (m_0^\nu)^2 \, \varepsilon^{2\mathcal{A}+m'+n'}(1 + \zeta^2 \varepsilon^{-m'}) \; , \; \Delta m_\odot^2 = \Delta m_{12}^2 = 0 \qquad (21)$$

The leptonic mixing matrix $U_l^0 = V_l^\dagger(\phi)V_\nu(\omega)$ is given by

$$U_l^0 \;=\; \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{\cos(\phi+\omega)}{\sqrt{2}} & -\frac{\cos(\phi+\omega)}{\sqrt{2}} & \sin(\phi+\omega) \\ \frac{\sin(\phi+\omega)}{\sqrt{2}} & \frac{\sin(\phi+\omega)}{\sqrt{2}} & \cos(\phi+\omega) \end{pmatrix} \qquad (22)$$

The above results exhibit a number of interesting properties of the model, that are worth mentioning at this point. We first observe that the model predicts an inverted neutrino mass hierarchy, since the smallest eigenvalue corresponds to m_{ν_3}. We further point out that the $U(1)_X$ symmetry implies large mixing effects in the neutrino mass matrix, in contrast to the situation of the charged fermion sector where the mixing is small. Moreover, at this level of approximation, a *zero*-entry for the element U_{13} is predicted in the mixing matrix. The rest of the elements are determined by two angles, ϕ arising from the charged lepton mass matrix diagonalisation and ω arising from the neutrino mass matrix.

In what follows, we will show how the above scenario is implemented, working out specific cases with the aim to find explicit sets of $U(1)_X$ charges which interpret the neutrino data. For the specific solutions we have set m' even and n' odd. Then, from the formulae of Table (2), we find that the leptons have fractional $U(1)_X$-charges of the form $\frac{2k+1}{4}$, with k integer.

Choosing for example, the values $m = 4, n = 8, m' = 2, n' = 7, h_1 = 2, h_2 = 0, A = -\frac{5}{2}$ we obtain the charge assignments of solution A of Table 3 and the following fermion mass

matrices for the quarks

$$M_{u,d} \sim m_0^{u,d} \begin{pmatrix} \varepsilon^8 & \varepsilon^6 & \varepsilon^4 \\ \varepsilon^6 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1 \end{pmatrix}. \tag{23}$$

(which is the texture discussed in [7]), the charged leptons

$$M_e \sim m_0^e \begin{pmatrix} \delta\,\varepsilon^7 & 0 & 0 \\ 0 & \varepsilon^2 & a\,\varepsilon \\ 0 & a\,\varepsilon & 1 \end{pmatrix} \tag{24}$$

and the neutrinos

$$M_\nu^0 \sim m_0^\nu \begin{pmatrix} 0 & -\varepsilon^2 & \zeta\,\varepsilon \\ -\varepsilon^2 & 0 & 0 \\ \zeta\,\varepsilon & 0 & 0 \end{pmatrix}. \tag{25}$$

Charged lepton masses can be fit within a range of the mass matrix parameters in (24). For example, choosing $\varepsilon \sim 0.28$, $\alpha \sim -1.3$ and $\delta \sim 2$, the correct mass spectrum is obtained. Atmospheric neutrino oscillation mass-squared difference is then reproduced for $M \sim 5 \times 10^{13} GeV$ modulo order one coefficients. This scale is quite low to be identified with the string scale in heterotic constructions, it is however compatible with type I superstring models where the string scale is tight to the Planck scale. Other configurations of additional $U(1)_X$-charges are also possible since the mass matrices under consideration do not depend on the parameters q_3, ℓ_3. For example choosing solution B of Table 3 we obtain the same mass matrices as in solution A considered above.

As already noted however, at this level of analysis, the neutrino mass splitting between the first and second generation does not appear because the two eigenstates are degenerate. Moreover, the solar neutrino mixing angle is maximal, a situation disfavored by recent data. This discrepancy can be lifted however, if additional non-zero entries are generated by hierarchically smaller effects. For example, if we assume an additional pair of singlet fields $\chi, \bar\chi$ with $U(1)_X$-charges $\pm 3/2$ we obtain $M_{\nu_{23}} = M_{\nu_{32}} \propto \eta$ and $M_{\nu_{11}} \propto \bar\eta^3$ (the appearance of several singlet fields is a usual phenomenon in string models). It is also possible to generate the required entries by some other mechanism, for example supersymmetry breaking. We find it interesting that two additional entries, for example 11 and 23, smaller than the entries 12 and 13 already present at this level, would be sufficient to bring the final form of the neutrino matrix to an acceptable two-zero texture mass matrix [19], that provides the necessary mass splitting and interpret accurately the experimental data. To show that this is indeed the case, let us assume that, after the inclusion of these effects and in the basis where the charged lepton mass matrix is diagonal, the neutrino mass matrix takes the form

$$M_\nu = m_0^\nu \begin{pmatrix} 2x & -\cos\bar\omega & \sin\bar\omega \\ -\cos\bar\omega & 0 & 2y \\ \sin\bar\omega & 2y & 0 \end{pmatrix} \tag{26}$$

where $\bar{\omega} = \omega + \phi$. Using the above stated assumption that $x, y < \cos \bar{\omega}, \sin \bar{\omega}$ the eigenvalues of (26) are

$$m_{\nu_1} \approx m_0^\nu \left(-1 + x - y \sin (2\bar{\omega})\right) \tag{27}$$

$$m_{\nu_2} \approx m_0^\nu \left(1 + x - y \sin (2\bar{\omega})\right) \tag{28}$$

$$m_{\nu_3} \approx 2\, m_0^\nu\, y\, \sin (2\bar{\omega}) \tag{29}$$

where higher order corrections $\mathcal{O}(x^2, xy, y^2)$ etc, are omitted. In terms of the above eigenvalues, the mass-squared differences are

$$\Delta m_{12}^2 \approx 4m_0^2(x - y \sin(2\bar{\omega})) \tag{30}$$

$$\Delta m_{23}^2 \approx m_0^2 \left(1 - 2(x - y \sin(2\bar{\omega}))\right) \tag{31}$$

The diagonalising matrix is then

$$U_l \approx \begin{pmatrix} \frac{1}{\sqrt{2}} + \frac{x+y\,\sin(2\,\bar{\omega})}{2\sqrt{2}} & \frac{1}{\sqrt{2}} - \frac{x+y\sin(2\,\bar{\omega})}{2\sqrt{2}} & 2y\cos(2\,\bar{\omega}) \\ -\frac{\cos(\bar{\omega})}{\sqrt{2}} + g(x,y) & \frac{\cos(\bar{\omega})}{\sqrt{2}} + g(x,y) & \sin(\bar{\omega}) \\ \frac{\sin(\bar{\omega})}{\sqrt{2}} - f(x,y) & -\frac{\sin(\bar{\omega})}{\sqrt{2}} - f(x,y) & \cos(\bar{\omega}) \end{pmatrix} \tag{32}$$

where again we have omitted higher order corrections and $f(x,y), g(x,y)$ in the matrix entries stand for

$$f(x,y) = \frac{(5\,y\,\cos(\bar{\omega}) + 3\,y\,\cos(3\,\bar{\omega}) + 2\,x\,\sin(\bar{\omega}))}{4\sqrt{2}} \tag{33}$$

$$g(x,y) = -\frac{(2\,x\,\cos(\bar{\omega}) + 5\,y\,\sin(\bar{\omega}) - 3\,y\,\sin(3\,\bar{\omega}))}{4\sqrt{2}} \tag{34}$$

Identifying the entries of the diagonalising matrix (32) with the standard parametrization angles θ_{ij} $(\sin \theta_{ij} \equiv s_{ij}, \cos \theta_{ij} \equiv c_{ij})$,

$$U_\nu = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \tag{35}$$

up to order $\mathcal{O}(x^2, xy, y^2)$ corrections (assuming the CP-phase $\delta = 0$) we find

$$\tan \theta_{23} \approx \tan \bar{\omega} \tag{36}$$

$$\tan \theta_{13} \approx 2y \cos(2\bar{\omega}) \tag{37}$$

$$\tan \theta_{12} \approx 1 - (x + y \sin(2\bar{\omega})) \tag{38}$$

Here, $\tan \theta_{23}$ differs from the original $\tan(\bar{\omega})$ only up to second order corrections on the parameters x, y, while $\tan \theta_{12}$'s value depends on the linear combination $(x + y \sin(2\bar{\omega}))$ of x and y. On the other hand, the neutrino mass-squared differences have a ratio which depends on a different x, y linear combination, $(x - y \sin(2\bar{\omega})$,

$$\frac{\Delta m_{12}^2}{\Delta m_{23}^2} = \frac{4(x - y \sin(2\bar{\omega}))}{1 - 2(x - y \sin(2\bar{\omega}))} \tag{39}$$

We note that it is crucial that two different combinations of x, y enter in the expressions (38) and (39). Clearly, if either of x, y is taken zero, the data cannot be reconciled.

The allowed ranges for mixing angles at 3σ are given by,

$$0.29 \leq \tan^2 \theta_{12} \leq 0.64 \,, \tag{40}$$

$$0.31 \leq \sin^2 \theta_{23} \leq 0.72 \,, \tag{41}$$

$$\sin^2 \theta_{13} < 0.054$$

and the mass-squared differences have the range

$$5.4 \times 10^{-5} \leq \Delta m_{12}^2 / eV^2 \leq 9.5 \times 10^{-5}, \quad 1.4 \times 10^{-3} \leq \Delta m_{23}^2 / eV^2 \leq 3.7 \times 10^{-3}$$

Using relations (38), (39) and the experimental data we find that experimentally acceptable $\tan \theta_{12}$ values can be satisfied for $x \approx [0.10 - 0.24]$ and $y \approx [0.10 - 0.22]$, assuming $\bar{\omega}$ to be maximal. We remark that these values in a wide portion of the acceptable range, are sufficiently smaller that the order one 12- and 13-neutrino mass matrix entries and thus our approximation is consistent. In this case we also have $\theta_{13} = 0$. Departing slightly from $\bar{\omega} = \pi/4$ we still have acceptable values for x, y and θ_{13} within the experimental limits, however the parameter space consistent with our approximation is reduced. These facts justify the assumption that the 11, 23 elements can be generated perturbatively. We finally check the impact of the above on the parameter related to $\beta\beta_{0\nu}$-decay effective neutrino mass. Ignoring the tiny m_{ν_3} contribution and second order effects, this is given by

$$|\langle m_{ee} \rangle| \approx 2 \, m_0 \, y \, \sin 2\bar{\omega} \tag{42}$$

which is one order of magnitude below the current experimental bound for the parameter region where our perturbative approach is valid.

4. Conclusions

In this letter, we have presented a simple extension of the Minimal Supersymmetric Standard model by an anomalous $U(1)_X$ symmetry broken at some high scale M and attempted to interpret the recent neutrino experimental data using just the left-handed neutrino components. Assuming symmetric mass matrices and that the third generation of up, down quarks and charged fermions acquire masses at tree-level, we derive the general charge assignments for MSSM fermions and examine their implications for the Majoranna neutrino mass matrix resulting from the dimension 5 operator $(LH)^2/M$. We find that the model leads naturally to inverted mass hierarchy for neutrinos, $\theta_{13} = 0$ and maximal atmospheric mixing for $M \sim 10^{13-14} GeV$. At this level the absolute masses of the lightest eigenstates are equal and solar neutrino mixing turns out to be also maximal. We show that higher order corrections which may arise from supersymmetry breaking or additional singlet fields, lift the mass degeneracy and the solar neutrino data can be accurately described. We derive explicit fermion $U(1)_X$ charge assignments that realize the above scenario.

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